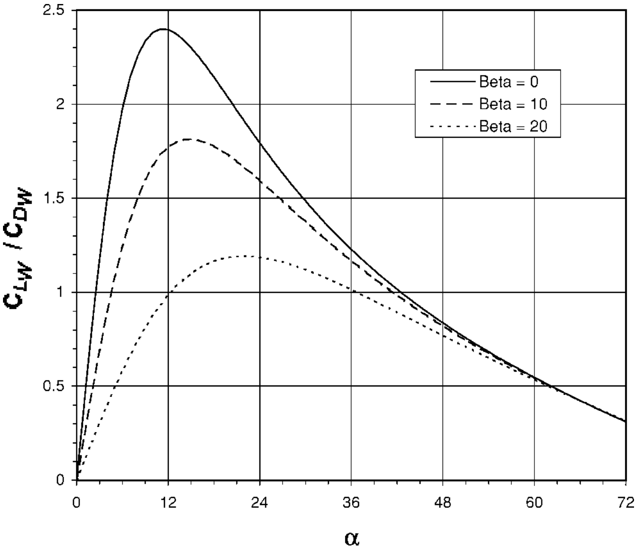

Fig. 3 Large-α lift-to-drag ratio vs α for $\beta = 0$, 10, and 20 deg.

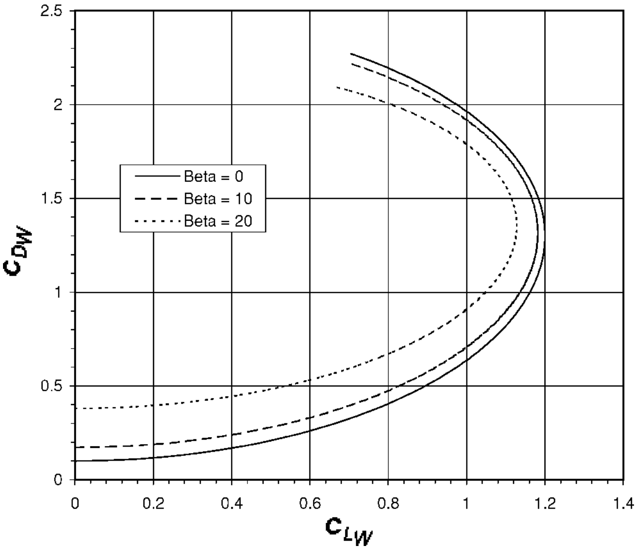

Fig. 4 Large-α drag polar for $\beta = 0$, 10, and 20 deg.

revolution is guided with equal effectiveness via a bank-to-turn or yaw-to-turn strategy.

Conclusions

Aeroforce coefficients for large α and β may be modeled via trigonometric functions that provide analytical insight and computational efficiency, benefit derivation of guidance laws, and could aid predictions of flight vehicle performance, particularly for aircraft and missiles that are maneuverable at large aeroangles.

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Two-Timescale Analysis of Phugoid Mode

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Introduction

SINCE the first Phugoid approximation derived by Lanchester,[1] literal approximations of aircraft modes have been of great interest to the aeronautical community. Classical approximations based on the hypothesis of complete and partial separation between fast and slow modes are well described by McRuer et al.[2] Earlier attempts to describe longitudinal modes based on singular perturbation theory have been proposed by Xu[3] and Khalil and Chen.[4] A recent approximation of longitudinal modes was described by Kamesh and Pradeep,[5,6] while further approximations based on the difference between the two timescales are given by Ananthkrishnan and Unnikrishnan[7] and Ananthkrishnan and Ramadevi.[8]

The reason why literal approximations of longitudinal modes are still of interest to flight dynamicists[9,10] can be found in the fact that while short-period parameters and Phugoid frequency are well estimated by classical simplified models, Phugoid damping approximation can still be improved.

In the present work we present an analytical procedure for Phugoid-mode literal approximation by applying different timescale analysis techniques. With respect to classical approximations, no assumption on separation between aircraft modes is made, and under the hypothesis of two-timescales property a change of variable is introduced, allowing the block diagonalization of the system into two slow and fast subsystems. The slow subsystem provides the new Phugoid-mode approximation. Numerical results show that the classical approximations of Phugoid damping are strongly improved.

The rest of the paper is organized as follows. First, the linearized aircraft dynamics are summarized and the proposed approximation based on the two-timescale technique is developed. Then, considerations on numerical results and conclusions are presented.

Longitudinal Aircraft Dynamics

For the purposes of this paper, we refer to a linearized set of equations about a steady flight trim condition. It is assumed that 1) $X_{\dot{w}}$, X_q, $Z_{\dot{w}}$, and Z_q stability derivatives are zero; and 2) the flight path of the aircraft is horizontal: $\gamma_0 = 0$. Under the preceding assumptions, the longitudinal equations of motions referred to a stability axis reference frame are

$$\dot{u} = X_u u - g\theta + X_w w, \qquad \dot{\theta} = q$$

$$\dot{w} = Z_u u + Z_w w + U_0 q$$

$$\dot{q} = M_u u + M_w w + M_{\dot{w}} \dot{w} + M_q q \tag{1}$$

Using standard notations,[2] the general polynomial form for the longitudinal aircraft dynamics is

$$\Delta_{\text{long}} = As^4 + Bs^3 + Cs^2 + Ds + E$$

$$= \left(s^2 + 2\xi_{\text{sp}}\omega_{\text{sp}} s + \omega_{\text{sp}}^2\right)\left(s^2 + 2\xi_p \omega_p s + \omega_p^2\right) \tag{2}$$

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Literal expressions for the coefficients of the preceding equation, in terms of stability derivatives, are

$$A = 1, \qquad B = 2\xi_{sp}\omega_{sp} + 2\xi_p\omega_p = -M_q - M_{\dot{w}}U_0 - Z_w - X_u$$

$$C = \omega_{sp}^2 + \omega_p^2 + 2\xi_{sp}\omega_{sp}2\xi_p\omega_p = Z_w M_q - M_w U_0 - X_w Z_u + X_u(M_q + M_{\dot{w}}U_0 + Z_w)$$

$$D = (2\xi_{sp}\omega_{sp})\omega_p^2 + (2\xi_p\omega_p)\omega_{sp}^2 = -X_u(Z_w M_q - M_w U_0) + Z_u(X_w M_q + gM_{\dot{w}}) - M_u(X_w U_0 - g)$$

$$E = \omega_{sp}^2\omega_p^2 = g(Z_u M_w - Z_w M_u) \tag{3}$$

Timescale Analysis

Starting from the system (1), literal approximations for aircraft longitudinal modes are based on the assumption that the two couples of eigenvalues have a different timescale. This allows one to distinguish between the fast mode (short period) and the slow mode (Phugoid). The short-period mode, which is characterized by high frequency and damping values, describes the rotations of aircraft about its center of mass, whereas the Phugoid mode, with light damping and low frequency, represents the translatory motion of the center of mass. Let us now rewrite the longitudinal dynamics (1) in the classical state-space form:

$$\begin{bmatrix} \dot{\bar{u}} \\ \dot{\theta} \\ \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} X_u & -g/U_0 & X_w & 0 \\ 0 & 0 & 0 & 1 \\ Z_u & 0 & Z_w & 1 \\ M_u^* U_0 & 0 & M_w^* U_0 & M_q^* \end{bmatrix} \begin{bmatrix} \bar{u} \\ \theta \\ \alpha \\ q \end{bmatrix} \tag{4}$$

where

$$M_u^* = M_u + M_{\dot{w}}Z_u, \qquad M_w^* = M_w + M_{\dot{w}}Z_w$$

$$M_q^* = M_q + M_{\dot{w}}U_0$$

are the $\dot{\alpha}$-corrected pitching moments derivatives, $\alpha = w/U_0$ is the angle of attack, and $\bar{u} = u/U_0$ is the normalized forward velocity.

Classical Phugoid-mode approximations that are available in most textbooks are based on the assumption of complete or partial separation between Phugoid and short-period modes. With the present approach the assumption of complete or partial separation is removed. This means that when Phugoid state variables start to vary, the short-period state variables are free to vary.

Let us now rewrite the unforced longitudinal dynamics in the two time-scale standard form:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}_{11}\boldsymbol{x} + \boldsymbol{A}_{12}\boldsymbol{z}, \qquad \dot{\boldsymbol{z}} = \boldsymbol{A}_{21}\boldsymbol{x} + \boldsymbol{A}_{22}\boldsymbol{z} \tag{5}$$

where $\boldsymbol{x} = [\bar{u} \;\; \theta]'$ and $\boldsymbol{z} = [\alpha \;\; q]'$ are the vectors containing the slow and the fast variables respectively, and

$$\boldsymbol{A}_{11} = \begin{bmatrix} X_u & -g/U_0 \\ 0 & 0 \end{bmatrix}, \qquad \boldsymbol{A}_{12} = \begin{bmatrix} X_w & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boldsymbol{A}_{21} = \begin{bmatrix} Z_u & 0 \\ M_u^* U_0 & 0 \end{bmatrix}, \qquad \boldsymbol{A}_{22} = \begin{bmatrix} Z_w & 1 \\ M_w^* U_0 & M_q^* \end{bmatrix}$$

If the system described by Eq. (5) has the two-timescale property, it can be decoupled into two slow and fast subsystems.[11] To this aim, let us introduce the following change of variable:

$$\begin{bmatrix} \boldsymbol{x}_s \\ \boldsymbol{z}_f \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} - \boldsymbol{ML} & -\boldsymbol{M} \\ \boldsymbol{L} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} \tag{6}$$

where $\boldsymbol{L}$ and $\boldsymbol{M}$ matrices are the solutions of the following nonlinear algebraic equations:

$$\boldsymbol{A}_{12} - \boldsymbol{A}_{22}\boldsymbol{L} + \boldsymbol{L}\boldsymbol{A}_{11} - \boldsymbol{L}\boldsymbol{A}_{12}\boldsymbol{L} = 0 \tag{7}$$

$$(\boldsymbol{A}_{11} - \boldsymbol{A}_{12}\boldsymbol{L})\boldsymbol{M} - \boldsymbol{M}(\boldsymbol{A}_{22} + \boldsymbol{L}\boldsymbol{A}_{12}) + \boldsymbol{A}_{12} = 0 \tag{8}$$

while $\boldsymbol{I}$ is the identity matrix.

The system (5) is decoupled into the two fast and slow subsystems:

$$\begin{bmatrix} \dot{\boldsymbol{x}}_s \\ \dot{\boldsymbol{z}}_f \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_s & 0 \\ 0 & \boldsymbol{A}_f \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_s \\ \boldsymbol{z}_f \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_{11} - \boldsymbol{A}_{12}\boldsymbol{L} & 0 \\ 0 & \boldsymbol{A}_{22} + \boldsymbol{L}\boldsymbol{A}_{12} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_s \\ \boldsymbol{z}_f \end{bmatrix} \tag{9}$$

If $\boldsymbol{A}_{22}$ is nonsingular, the matrix $\boldsymbol{L}$ can be approximated as follows[12]:

$$\boldsymbol{L}_i = \boldsymbol{A}_{22}^{-1}\boldsymbol{A}_{21} + \boldsymbol{A}_{22}^{-1}\boldsymbol{L}_{i-1}(\boldsymbol{A}_{11} - \boldsymbol{A}_{12}\boldsymbol{L}_{i-1}), \qquad \boldsymbol{L}_0 = \boldsymbol{A}_{22}^{-1}\boldsymbol{A}_{21} \tag{10}$$

The existence of a solution to the matrix equality (7) and the validity of its approximation is stated in Kokotovic et al.[12] by Lemma 2.1 and Lemma 2.2.

The physical meaning of this procedure can be better appreciated by obtaining $\boldsymbol{L}$ with a process of iterative residualization.

Let introduce the hypothesis that Phugoid and short-period modes are completely separated. This means that when the Phugoid variables u and θ start to vary the short-period variables have reached their steady-state values. This allows one to consider $\dot{\alpha} = \dot{q} = 0$.

Accordingly, the system (5) becomes

$$\dot{\boldsymbol{x}} = \boldsymbol{A}_{11}\boldsymbol{x} + \boldsymbol{A}_{12}\boldsymbol{z}, \qquad 0 = \boldsymbol{A}_{21}\boldsymbol{x} + \boldsymbol{A}_{22}\boldsymbol{z} \tag{11}$$

from the second equation the static residual is evaluated as

$$\boldsymbol{z}^* = -\boldsymbol{A}_{22}^{-1}\boldsymbol{A}_{21}\boldsymbol{x} \tag{12}$$

The use of the preceding equation allows to obtain the following slow model equation:

$$\dot{\boldsymbol{x}} = \left(\boldsymbol{A}_{11} - \boldsymbol{A}_{12}\boldsymbol{A}_{22}^{-1}\boldsymbol{A}_{21}\right)\boldsymbol{x} = \boldsymbol{A}_s^{(0)}\boldsymbol{x} \tag{13}$$

This procedure is the same described by Ananthkrishnan and Unnikrishnan,[7] and the last result is identical to that obtained from the system (9) by choosing $\boldsymbol{L} = \boldsymbol{L}_0$ (Ref. 4). Phugoid-mode frequency and damping expression can be easily carried out:

$$\omega_p^2 = \det\left[\boldsymbol{A}_s^{(0)}\right] = g\frac{M_w^* Z_u - Z_w M_u^*}{Z_w M_q^* - M_w^* U_0} \tag{14}$$

$$2\omega_p\xi_p = -\text{trace}\left[\boldsymbol{A}_s^{(0)}\right] = -X_u + X_w\frac{M_q^* Z_u - M_u^* U_0}{Z_w M_q^* - M_w^* U_0} \tag{15}$$

This result is known in the literature as classical two-degree-of-freedom Phugoid approximation. The contribution of the present paper is to improve the preceding approximation by deriving an additional residual, which can be called first dynamic residual. Let us extend the system (5) by adding the new state variable $\boldsymbol{w} = \dot{\boldsymbol{z}}$:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}_{11}\boldsymbol{x} + \boldsymbol{A}_{12}\boldsymbol{z}, \qquad \dot{\boldsymbol{z}} = \boldsymbol{A}_{21}\boldsymbol{x} + \boldsymbol{A}_{22}\boldsymbol{z}$$
$$\dot{\boldsymbol{w}} = \boldsymbol{A}_{21}(\boldsymbol{A}_{11}\boldsymbol{x} + \boldsymbol{A}_{12}\boldsymbol{z}) + \boldsymbol{A}_{22}\boldsymbol{w} \tag{16}$$

Here the hypothesis of complete separation between the two longitudinal modes is removed. This means that during the slow variables transient the fast variables are free to slowly vary. This allows one to assume that $\dot{\boldsymbol{w}} = 0$. Solving for $\boldsymbol{w}$ in the third row of system (16) and using Eq. (12) leads to

$$\boldsymbol{w} = \dot{\boldsymbol{z}} = -\boldsymbol{A}_{22}^{-1}\boldsymbol{A}_{21}\left(\boldsymbol{A}_{11} - \boldsymbol{A}_{12}\boldsymbol{A}_{22}^{-1}\boldsymbol{A}_{21}\right)\boldsymbol{x} \tag{17}$$

This is an improvement over the assumption of $\dot{\boldsymbol{z}} = 0$ taken for deriving the static residual. The first dynamic residual for the $\boldsymbol{z}$ variable is obtained by substituting the preceding equation into the second row of system (16):

$$-\boldsymbol{A}_{22}^{-1}\boldsymbol{A}_{21}\left(\boldsymbol{A}_{11} - \boldsymbol{A}_{12}\boldsymbol{A}_{22}^{-1}\boldsymbol{A}_{21}\right)\boldsymbol{x} = \boldsymbol{A}_{21}\boldsymbol{x} + \boldsymbol{A}_{22}\boldsymbol{z} \tag{18}$$

and then

$$z^{**} = -A_{22}^{-1}\left[A_{21} + A_{22}^{-1}A_{21}\left(A_{11} - A_{12}A_{22}^{-1}A_{21}\right)\right]x \tag{19}$$

Finally, using the preceding value of z^{**} allows one to obtain the improved model for the slow mode

$$\dot{x} = \left[A_{11} + A_{12}A_{22}^{-1}\left(A_{21} + A_{22}^{-1}A_{21}A_s^{(0)}\right)\right]x = A_s^{(1)}x \tag{20}$$

which is identical to that obtained from the system (9) by choosing $i = 1$ in Eq. (10). By applying the preceding discussion, the new literal approximations for Phugoid-mode frequency and damping are

$$\omega_p^2 = \det\left[A_s^{(1)}\right] = g\frac{M_w^* Z_u - Z_w M_u^*}{Z_w M_q^* - M_w^* U_0} + gX_w\frac{\left[M_q^*\left(M_u^* U_0 - M_q^* Z_u\right) + U_0\left(M_u^* Z_w - M_w^* Z_u\right)\right]}{\left(Z_w M_q^* - M_w^* U_0\right)^3} \tag{21}$$

$$2\omega_p\xi_p = -\mathrm{trace}\left[A_s^{(1)}\right] = -X_u + X_w\frac{M_q^* Z_u - M_u^* U_0}{Z_w M_q^* - M_w^* U_0} + X_w\frac{Z_u\left(M_w^* U_0 + M_q^{*2}\right) - M_u^* U_0\left(Z_w + M_q^*\right)}{\left(Z_w M_q^* - M_w^* U_0\right)^2} \times \left[X_u - \frac{X_w\left(M_q^* Z_u - M_u^* U_0\right)}{Z_w M_q^* - M_w^* U_0}\right] + g\frac{M_w^* Z_u\left(Z_w + M_q^*\right) - M_u^*\left(Z_w^2 + M_w^* U_0\right)}{\left(Z_w M_q^* - M_w^* U_0\right)^2} \tag{22}$$

Results

Numerical computations are carried out using Boeing 747 aircraft data,[13] and the values for Phugoid-mode frequency and damping of the new approximation are compared with the results of static residualization and with the approximations found in the recent literature.

Tables 1 and 2 show the comparison between the static residual approximation described by Eqs. (14) and (15) and the new approximation. It is clear that the proposed model strongly improves the previous Phugoid damping approximation, whereas variations on Phugoid frequency approximations are not relevant.

Table 3 shows the comparison between the recent approximation proposed by Ananthkrishnan and Ramadevi[8] and the first dynamic approximation. Such a comparison is made by neglecting the effect of $M_{\dot{w}}$ derivative. The corresponding model is derived by simply imposing $M_u^* = M_u$, $M_w^* = M_w$, and $M_q^* = M_q$ in Eqs. (21) and (22).

The two-timescale approximation replaces the results obtained by Kamesh and Pradeep.[5] (See Table 4.) The advantage is that the

Table 1 Phugoid-mode frequency: first dynamic residual vs static residual

Flight condition	Exact value	Static residual		First dynamic residual	
		Approximate	Error, %	Approximate	Error, %
1	0.1517	0.1527	−0.68	0.1522	−0.37
2	0.1267	0.1264	−0.24	0.1261	0.47
3	0.0753	0.0748	0.67	0.0748	0.72
4	0.0368	0.0366	0.54	0.0367	0.49
5	0.0823	0.0819	0.54	0.0818	0.66
6	0.0653	0.0649	0.47	0.0649	0.52
7	0.0102	0.0102	0.38	0.0102	0.29
8	0.0782	0.0775	0.81	0.0776	0.71
9	0.0673	0.0670	0.46	0.0669	0.56
10	0.0310	0.0310	0.31	0.0310	0.31

Table 2 Phugoid-mode damping: first dynamic residual vs static residual

Flight condition	Exact value	Static residual		First dynamic residual	
		Approximate	Error, %	Approximate	Error, %
1	0.0416	0.1808	−335.21	0.0435	−4.68
2	0.0225	0.1053	−367.45	0.0238	−5.66
3	0.0319	0.0604	−89.95	0.0324	−1.57
4	0.1099	0.1528	−39.01	0.1101	−0.15
5	0.0240	0.0488	−103.09	0.0246	−2.15
6	0.0264	0.0477	−80.21	0.0268	−1.21
7	0.3103	0.5043	62.52	0.3114	−0.33
8	0.0637	0.0585	8.19	0.0641	−0.70
9	0.0489	0.0453	7.33	0.0492	−0.65
10	0.3044	0.3518	−15.58	0.3057	−0.44

Table 3 Phugoid-mode damping: first dynamic residual (without $M_{\dot{w}}$) vs recent approximation

Flight condition	Exact value	Ananthkrishanan and Ramadevi[8]		First dynamic residual	
		Approximate	Error, %	Approximate	Error, %
1	0.0416	0.0683	−64.3101	0.0381	8.3130
2	0.0225	0.0377	−67.2629	0.0210	6.6454
3	0.0319	0.0374	−17.3402	0.0315	1.1230
4	0.1099	0.1173	−6.7406	0.1064	3.1890
5	0.0240	0.0350	−45.7507	0.0243	−1.1963
6	0.0264	0.0335	−26.6196	0.0263	0.4846
7	0.3103	0.3155	−1.6769	0.2931	5.5547
8	0.0637	0.0747	−17.2749	0.0646	−1.4761
9	0.0489	0.0579	−18.5627	0.0497	−1.7586
10	0.3044	0.3069	−0.8486	0.3042	0.0503

Table 4 Phugoid-mode damping: first dynamic residual (with $M_{\dot{w}}$) vs recent approximation

Flight condition	Exact value	Kamesh and Pradeep[5]		First dynamic residual	
		Approximate	Error, %	Approximate	Error, %
1	0.0416	0.0445	−6.9885	0.0435	−4.6755
2	0.0225	0.0242	−7.5616	0.0238	−5.6562
3	0.0319	0.0324	−1.7087	0.0324	−1.5739
4	0.1099	0.1100	−0.0595	0.1101	−0.1508
5	0.0240	0.0247	−2.6232	0.0246	−2.2514
6	0.0264	0.0268	−1.3480	0.0268	−1.2111
7	0.3103	0.3107	−0.1212	0.3114	−0.3331
8	0.0637	0.0641	−0.6149	0.0641	−0.6958
9	0.0489	0.0492	−0.7320	0.0492	−0.6477
10	0.3044	0.3057	−0.4388	0.3057	−0.4389

slow model obtained with the proposed procedure is an effective Phugoid-mode dynamics, involving only the slow variables. This fact allows the control system designer to work with a suitable, but order-reduced, model.

Conclusions

Classical Phugoid damping approximations for the longitudinal dynamics are inaccurate. In this Note an analytical procedure based on different timescale analysis was used to obtain a new two degrees-of-freedom Phugoid model.

Because of a block diagonalization of the state matrix, a decoupled system made of two slow and fast subsystems is obtained with the only assumption of the two-timescale property for the longitudinal dynamics.

The most important result is that the proposed Phugoid model is accurate. Numerical results show that errors are small in all of the flight conditions considered. Moreover, the proposed Phugoid model improves classical approximation available in most textbooks and the other approximations proposed in the recent literature. The only lack of such an approximation is that the complexity of literal expressions tends to obscure the fundamental effects of aerodynamic stability derivatives on the Phugoid mode.

Finally, a complete state-space model based only on slow variables is available. This can be helpful in the synthesis of control systems.

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